Moral Hazard

The Principal-Agent Problem

- We consider interactions between two agents: a *principal* and an *agent*.
- The principal has the bargaining power and designs a *contract*.
- The agent reacts to the principal's contract and chooses an *action* or a *message*.
- The principal is risk neutral, whereas the agent is risk averse.

Examples

- Landlord (P) vs. Tenant (A)
- Employer (P) vs. Employee (A)
- Insurer (P) vs. Insured (A)

Asymmetric Information

- The agent knows something that the principal cannot observe.
  - *Hidden Action*: P cannot observe A's action (Moral Hazard).
  - *Hidden Information*: P cannot observe A's type (Adverse Selection).
- Due to asymmetric information, the agent can extract a *rent*.
- Question: How can one minimize the agent's rent? What is the optimal contract chosen by the principal?

Basic Model

- There is a result $x \in X$ (e.g. sales, harvest). The result is always observed by both parties.
- The agent chooses an effort level e.
- The relation between effort and the result is not deterministic but stochastic (if deterministic, the principal could have deduced the effort from the result).
- Results are ordered $x_1 < x_2 < ... < x_n$.
- $\Pr[x = x_i] = p_i(e)$; suppose that $p_i(e) > 0$ for all i, e.

Utilities

- The principal obtains the result x and pays a wage w.
- Her utility is given by $B(x - w)$ with $B' > 0$ and $B'' < 0$. *We often assume $B'' = 0$; that is, the principal is risk neutral.*
- The agent's utility is separable in wage and effort:

  $$u(w, e) = u(w) - v(e),$$

  where $u' > 0, u'' < 0$, $v' > 0, v'' > 0$.
- Conflict of interests: the principal cares about the result but not the effort, whereas the agent cares about the effort but not the result.

Reservation Utilities, Participation and Contract

- The time sequence of the model is the following.

1. P offers a contract, specifying a wage for the agent as a function of observed variables.
2. A accepts or rejects the contract. If A rejects, receives a *reservation utility* $\overline{U}$.
3. A chooses effort.
4. Results are realized, and a wage is paid to A.

Contracts with Symmetric Information

- Suppose that information is symmetric; that is, P observes the effort of A.
- A contract specifies both a payment scheme $w(x_i)$ and an effort level e in order to maximize

  $$\sum_i p_i(e)B(x_i - w(x_i)),$$

  subject to the constraint

  $$\sum_i p_i(e)u(w(x_i)) - v(e) \geq \overline{U}.$$

  The constraint is called the *participation constraint* of the agent.

Optimal Risk Sharing

- The Lagrangian is

$$\mathcal{L} = \sum_i p_i(e)B(x_i - w(x_i)) - \lambda[\overline{U} - (\sum p_i(e)u(w(x_i)) - v(e)).$$

- Fix e. For any x_i, the wage $w(x_i)$ is chosen to maximize the Lagrangian, thus:

$$\frac{\partial \mathcal{L}}{\partial w(x_i)} = -p_i(e)B'(x_i - w(x_i)) + \lambda p_i(e)u'(w(x_i)) = 0.$$

- Hence, for any x_i,

$$\frac{B'(x_i - w(x_i))}{u'(w(x_i))} = \lambda.$$

- *Optimal risk sharing occurs where marginal utilities are equalized for the principal and the agent.*

Principal and Agent

- If P is risk neutral, B' is a constant, so $u'(w(x_i))$ is a constant; that is, $w(x_1) = w(x_2) = ... = w(x_n)$.
- The agent receives a *fixed wage*.
- If A is risk neutral, u' is a constant so $B'(x_i - w(x_i))$ is a constant; thus, $x_i - w(x_i)$ is a constant.
- The agent receives a wage $x_i - k$. This is equivalent to a *fixed franchise* paid to the principal.

Moral Hazard

- The moral hazard problem arises when the effort of the agent is not observed.
- For one, a farmer's effort is not observed but the harvest is.
- For another, a salesman's effort is not observed but sales are.
- Finally, the behavior of a driver is not observed but accidents are.

Incentive Constraint

- In addition to the participation constraint, there is an *incentive constraint*.
- If P wants to induce effort level e^* it must be that $$\sum p_i(e^*)u(W(x_i)) - v(e^*) \geq \sum p_i(e)u(W(x_i)) - v(e) \;\; \forall e.$$
- If there are only two effort levels, the incentive constraint is simple to write.
- If there are many effort levels, the condition becomes difficult to write!

Two Effort Levels, Two Results

- Suppose that there are two effort levels, e^H and e^L with costs $v(e^H) = c^H > c^L = v(e^L)$.
- There are two possible results x^L, x^H and the probabilities are given by

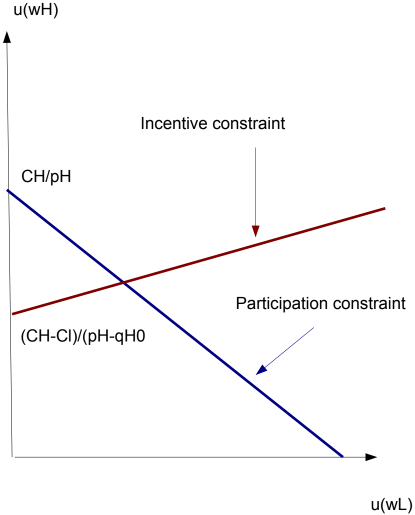
	x^H	x^L
e^H	p^H	p^L
e^L	q^H	q^L

- with $p^H > q^H$, $p^H + p^L = 1$, $q^H + q^L = 1$.

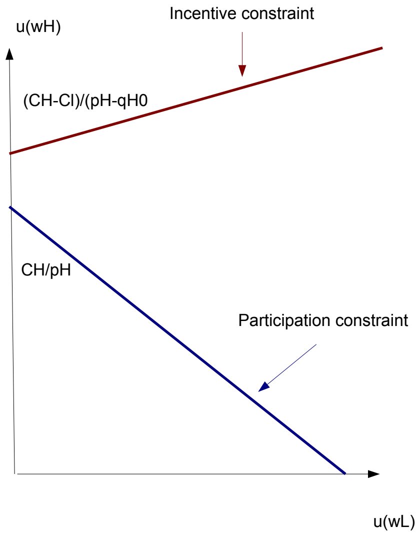
Participation and Incentive Constraints With e^H

- **Suppose that P wants to implement the high effort level.**
- The participation constraint is
$$p^H u(w^H) + p^L u(w^L) - c^H \geq \overline{U}.$$
- The incentive constraint is
$$p^H u(w^H) + p^L u(w^L) - c^H \geq q^H u(w^H) + q^L u(w^L) - c^L$$
or
$$(p^H - q^H) u(w^H) + (p^L - q^L) u(w^L) \geq (c^H - c^L).$$

Participation and Incentive Constraints (Cont.)



Participation and Incentive Constraints (Cont.)



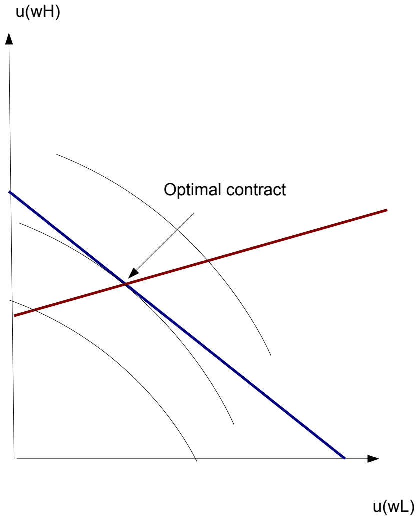
Principal's Profit

- The principal maximizes her profit
$$p^H(x^H - w^H) + p^L(x^L - w^L).$$

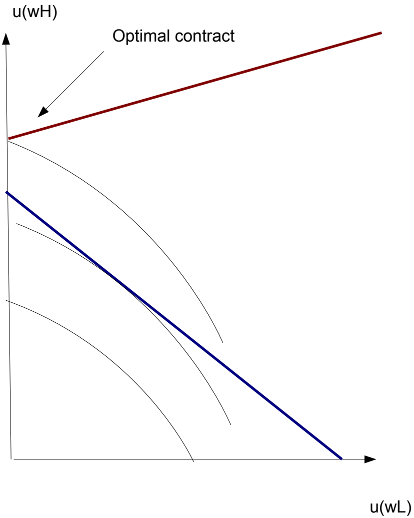
- We consider the isoprofit curves
$$p^H(x^H - w^H) + p^L(x^L - w^L) = \pi.$$

- These curves are decreasing curves in the space $(u(w^H), u(w^L))$ and they increase towards the southwest.

Optimal Contract



Optimal Contract (Cont.)



Interpretation

- The optimal contract always lies on the incentive constraint.
- The agent is sometimes pushed down to his reservation utility, sometimes not.

Mathematical Solution

- The Lagrangian is

$$\begin{aligned}\mathcal{L} &= p^H(x^H - w^H) + p^L(x^L - w^L)\\ &+ \lambda(\overline{U} - (p^H u(w^H) + p^L u(w^L) - c^H)\\ &+ \mu((c^H - c^L) - (p^H - q^H)u(w^H) - (p^L - q^L)u(w^L)).\end{aligned}$$

- The solution is

$$\begin{aligned}-1 - \lambda u'(w^H) - \mu(1 - \frac{q^H}{p^H})u'(w^H) &= 0,\\ -1 - \lambda u'(w^L) - \mu(1 - \frac{q^L}{p^L})u'(w^L) &= 0.\end{aligned}$$

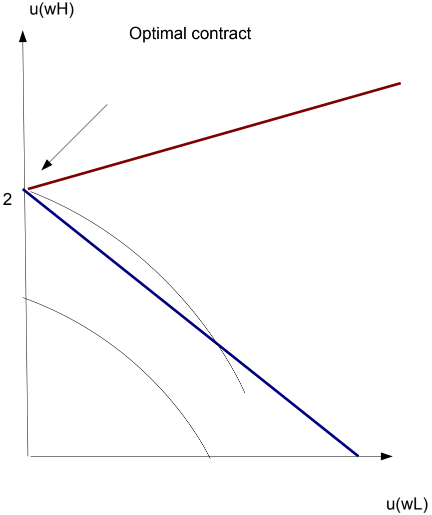
Interpretation of the Solution

- As $\mu \neq 0$, the optimal wages *are not constant*; that is $w^H \neq w^L$.
- In fact, as $\frac{q^H}{p^H} \leq \frac{q^L}{p^L}$, $w^H > w^L$.

Example

- Suppose $x^L = 0$, $x^H = 20$, $p^H = \frac{1}{2}$, $q^H = \frac{1}{4}$, $c^L = 0$, $c^H = \frac{1}{2}$.
- Suppose $u(w) = \sqrt{w}$, $\overline{U} = \frac{1}{2}$.
- The participation constraint is
$$\frac{1}{2}\sqrt{w_H} + \frac{1}{2}\sqrt{w_L} - \frac{1}{2} \geq \frac{1}{2} \to \sqrt{w_H} + \sqrt{w_L} \geq 2.$$
- The incentive constraint is
$$\frac{1}{4}\sqrt{w_H} - \frac{1}{4}\sqrt{w_L} \geq \frac{1}{2} \to \sqrt{w_H} = \sqrt{w_L} + 2.$$

Optimal Contract



Optimal Contract (Cont.)

- In the optimal contract, $w^H = 4$, $w^L = 0$.
- The profit of P is $\frac{1}{2}(20 - 4) = 8$.

Participation Constraint With e^L

- **Suppose that P wants to implement the low effort level.**

- The participation constraint is

$$\frac{1}{4}\sqrt{w_H} + \frac{3}{4}\sqrt{w_L} = \frac{1}{2}.$$

- As $w^H = w^L$, the wage is $w = \frac{1}{4}$, and the expected profit

$$\frac{20}{4} - \frac{1}{4} = \frac{19}{4}.$$

Other Models of Moral Hazard

- Severity of punishments.
- Giving bargaining power to the agent.
- Multiple agents: yardstick competition and using information on relative performance.
- Multiple principals: competition between principals and common agency.

Summary

- Consider a contract between two individuals: an agent and a principal.
- The principal proposes the contract, the agent accepts or rejects, and the contract is executed.
- The agent either has hidden information (adverse selection) or his effort is not observed (hidden action).
- In a symmetric information contract, the principal only faces a participation constraint, and risk is optimally shared between the principal and the agent.
- If the principal is risk neutral, the optimal contract is a fixed wage; if the agent is risk neutral, the optimal contract is equivalent to a fixed franchise paid to the principal.

Summary (Cont.)

- When the effort of the agent is not observed, the principal offers an incentive contract, where wages depend on the results.
- The principal faces both a participation and an incentive constraint.
- In the optimal contract, the incentive constraint is satisfied with equality; the agent may or may not be driven to his reservation utility.
- In the optimal contract, higher results are rewarded with higher wages.